A CRITICAL REVIEW OF DISPLACMENT CURRENT

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ABSTRACT

In the field of physical science, the concept of displacement current of Maxwell is considered as one of the most innovative ones. He developed his famous electromagnetic theory using this concept, without which modern scientific and technological development is simply not possible. In the Maxwell-Ampere law, time variation ofinstantaneous electric field, even in vacuum, has been identified as Displacement Current which is considered as another source of the magnetic field. But its direct experimental detection yet remains questionable, though several attempts have been taken so far. In this review article, effort is given to critically analyze the gap between its theoretical concept and experimental incompatibilities from different perspectives. Hertz's theoretical endeavour is followed to point out the reasons for criticism, which the importantconcept of displacement current received at its end.

Keywords: *Maxwell-Ampere law, displacement current, Hertz's Theory, electromagnetic field.*

1. Introduction

The 19th century great physicist Maxwell, considered to be the pillar of modern electrical sciencefor his electromagnetic theory, introduced the displacement current concept which having profound significance from scientific and philosophical points of view is considered to be closest toa very useful and genuine theory but built purely from speculations, by none other than Einstein [1]. Maxwell's equations, also known as Maxwell-Heaviside equations are a group of partial differential equations which form the core of electromagnetism and classical optics along with Lorentz force which additionally gives rise to the model for power generation, radar, wireless communication, electric motor, etc.[2].

Maxwellidentified the incompatibility of Ampere's circuital law when applied to an electric circuit with capacitors. This restricted applicability of the differential form of Ampere's circuital law [3] and to eliminate its inconsistency in notsatisfying the continuity equation of charge conservation and hence inapplicability for the time-varying field,Maxwell added the term of displacement current to modify Ampere's circuital law [4]. With thisproposed term, Maxwell deduced that every electric current (conduction as well as displacement) must form a closed circuit [5]. It is explained that electromotive force when acts on dielectric material, might produce polarization in it, even if it is vacuum, which results in displacement of the electricity in general, along a specific direction [6]. This variation of electric displacement constitutesall important displacement current.

Modified Ampere's law is applicable for any kind of loop including the loop enclosing surface between capacitor plate and its axis. During the charging of the capacitor, no conduction of charge exists between the plates. However, for the variation of charge accumulation on the plates with time, the resulting change of electric field results in producing the displacement current. The application of the modified version of Ampere's law is that all currents under consideration are closed and variation in electric field results in producing magnetic field.

But there is no significant direct experimental evidence to the electromagnetic action of currents in dielectrics due to the change of electric displacement and also there is extreme difficulty to reconcile the electromagnetic laws with the existence of electric currents which are not closed. Naturally, the question arises in accepting the existence of transient currents due to the variation of electric displacement and hence the field. At this point, Maxwell explained that the electric displacementrepresents basically the movement of electricity in the same sense asgeneral flow of electricity and hence only in a conductor a current is set up characteristic oftrue conduction[7]. So it may be noted here that any increase of displacement is equivalent to a current in positivedirection, whereas its diminution is equivalent to a current in the opposite direction. He also distinguished currents due to conduction effect and due to variation of displacement and viewed the electricity [8] as amobile , abstract and incompressible fluid, consisting of no electric charges.That's how Maxwell accommodated current in a dielectric material and clarified that in the charging capacitor circuit, total current would comprise the conduction current in the conducting wire and the displacement current in the dielectric medium even if it is vacuum[6].

2. Motivation behind the Study of Displacement Current

To search for itMaxwell's equations in vacuum are considered for electric field and magnetic field $\boldsymbol{E}$ and $\boldsymbol{B}$ respectively as

$$\boldsymbol{\nabla . E} = \frac{\boldsymbol{\rho}}{\boldsymbol{\varepsilon_0}} \text{(i)}$$

$$\boldsymbol{\nabla . B} = \mathbf{0} \text{(ii)}$$

$$\boldsymbol{\nabla X E} = -\frac{\boldsymbol{\partial B}}{\boldsymbol{\partial t}} \quad \text{(iii)}$$

$$\boldsymbol{\nabla X B} = \boldsymbol{\mu_0} \left(\boldsymbol{J} + \frac{\boldsymbol{\partial D}}{\boldsymbol{\partial t}}\right) \quad \text{(iv)}$$

where symbols represent their usual meaning.

Using the expression of scalar potential φ and vector potential **A**

$\boldsymbol{B} = (\boldsymbol{\nabla X A})$(v)
$\boldsymbol{E} = -\boldsymbol{\nabla}\varphi - \partial \boldsymbol{A}/\partial t$ (vi)

the inhomogeneous Maxwell's equationscan be written as [7]

- $\nabla^2\varphi - \frac{\partial}{\partial t}(\boldsymbol{\nabla . A}) = \frac{\boldsymbol{\rho}}{\boldsymbol{\varepsilon_0}}$from (i) and (vi)
and
- $\nabla^2\mathbf{A} + \boldsymbol{\nabla}(\boldsymbol{\nabla . A}) = \mu_0(\boldsymbol{J} + \frac{\boldsymbol{\partial D}}{\boldsymbol{\partial t}})$from (iv)
which explicitly show the displacement current.

Using Coulomb gauge condition for vector potential $\boldsymbol{\nabla . A} = 0$ in the above, the scalar potential equationchanges to
$\nabla^2\varphi = -\frac{\boldsymbol{\rho}}{\boldsymbol{\varepsilon_0}}$ **(a)**
⟹thePoisson's Eqn. with source of scalar potential as volume charge densityρ

and that for the vector potential
$\nabla^2\mathbf{A} = -\mu_0(\boldsymbol{J} + \frac{\boldsymbol{\partial D}}{\partial t})$ **(b)**

⟹Similar in form as Poisson's Eqn., but valid for the vector potential, with source term, representing combination of both the conduction current density and the unknown displacement current density [7].
But displacement current density term $\frac{\partial \boldsymbol{D}}{\partial \boldsymbol{t}}$ representing time variation of electric field ($\boldsymbol{\epsilon_0}$ $\boldsymbol{E}$) can never be considered as source term like $\boldsymbol{J}$, the conduction current density and ρ [9]. Also Maxwell's consideration was based on the Coulomb gauge for the potentials, though Maxwell's theory is gauge invariant. Another confusion arises to consider displacement current as a true current from different viewpoint. Using Biot-Savart's law, the magnetic field can easily be estimated for slowly varying fields without including the displacement current. Soquestion naturally arises that whether magnetic field can be produced from the displacement current for slowly varying fields [7] or not.

3. Hertz Theory andDisplacement Current

Twenty years after the publication of Maxwell's theory, Hertz's theory of electromagnetics was published which showed that how charges, currents and variation in fields caused the formulation of the electric and magnetic fields. The theory was found perfectly united when the restriction set on the current distribution of the original theory was removed. The principle of the unity of magnetic force and that of the electric forcecorrespond to the magnetic fields and the electricfields respectively, in Hertz's modern nomenclature. Few other distinctive features of this theoryincludes the principle of energy conservation, the action and reaction applied to closed circuitsystems, the laws of electromagnetic actions of closed current circuits [10]etc..

In the original work of Hertz, Ampere's law and Faraday's laws, stating the production of fields by electric current is considered as a separate set.According to Ampere's law,

$$\boldsymbol{\nabla X B} = \mu_0 \boldsymbol{J} \quad \text{(vii)}$$

which represents conduction (electric) current density $\boldsymbol{J}$ for a constant current flow and is a function of time. So

$$\boldsymbol{\nabla . J} = \boldsymbol{0} \text{(viii)}$$

Again, absence of magnetic monopole represented as

$$\boldsymbol{\nabla . B} = \boldsymbol{0}$$

holds for free space also. So following definition, an electric vector potential function, denoted by $\mathbf{A_e}$, can be introduced such that
$\boldsymbol{B}= \mu_0 \boldsymbol{H} = \boldsymbol{\nabla X A_e}$(ix)
Then from Eq. (vii)it is found
$\nabla^2\mathbf{A_e}= -\mu_0\boldsymbol{J}$(x)
assuming Coulomb Gauge for electric vector potential.

If electric current $\boldsymbol{I}$ is introduced such that [10]

$\mathbf{A_e}= \mu_0\boldsymbol{I}$
Eq. (ix) becomes $\boldsymbol{B} =\mu_0(\boldsymbol{\nabla X} \mathbf{I})$(xi)

and Eq. (x) is converted to
$\nabla^2\boldsymbol{I} = -\boldsymbol{J}$ (xii)

Now Hertz's induction law, representing time rate of variation of $\boldsymbol{I}$ must result in an electric field $\boldsymbol{E}$if

$$\boldsymbol{E}=-\boldsymbol{\mu_o}\frac{\partial \boldsymbol{I}}{\partial \boldsymbol{t}}\text{(xiii)}$$

which follows Faraday's laws of induction

$$\boldsymbol{\nabla} X\boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial \boldsymbol{t}}$$

Equations (xi) and (xiii) are considered by Hertz to represent magnetic and electric fields respectively.
Hertz proposed another set of equations based on the idea of magnetic current [10], which was considered to be the rate of change of magnetization (***M***), i.e.

$$\boldsymbol{J}_\mathbf{M} = -\mu_o \frac{\partial \boldsymbol{M}}{\partial \boldsymbol{t}} \quad \text{(xiv)}$$

Negative sign usein Eq. (xiv) represents the Ampere's law of magnetic origin in the same form as original Ampere's law, i.e.

$$\nabla X \boldsymbol{E} = \boldsymbol{J}_\mathbf{M} \quad \text{(xv)}$$

For the electric field produced by a magnetic current,

$\nabla . \boldsymbol{D} = \mathbf{0}$

where $\boldsymbol{D} = \epsilon_0 \boldsymbol{E},$

the electric displacement vectorin vacuum.

Corresponding magnetic vector potential, denoted by $\mathbf{A}_\mathbf{m}$can thus be introduced as

$\mathbf{D} = \epsilon_0 \boldsymbol{E} = \nabla \boldsymbol{X} \boldsymbol{A}_{\boldsymbol{M}}$(xvi),

like electric vector potential Eq.(ix).

Using Eqns. (xv) and (xvi) and considering $\nabla . \boldsymbol{A}_{\boldsymbol{M}} = \mathbf{0}$

$\nabla^2 \mathbf{A}_\mathbf{M} = -\epsilon_0 \boldsymbol{J}_\mathbf{M}$(xvii)

is obtained in analogy with Eq. (x).

Now if magnetic current $\boldsymbol{I}_{\boldsymbol{M}}$is introduced as

$\boldsymbol{I}_{\boldsymbol{M}} = \boldsymbol{A}_{\boldsymbol{M}} / \varepsilon_0$(xviii)

then from Eq. (xvi),relating magnetic current and vector potential it is found,

$$\boldsymbol{E} = \nabla \boldsymbol{X} \boldsymbol{I}_{\boldsymbol{M}} \quad \text{(xix)}$$

AgainHertz’s induction lawfor magnetic equivalence,representing time rate of variation of $\boldsymbol{I}_{\boldsymbol{M}}$ must result in corresponding field as [10]

$\mathbf{H} = \in_\mathbf{o} \frac{\partial \boldsymbol{I}_{\boldsymbol{M}}}{\partial \boldsymbol{t}}$ (xx)

and the magnetic Faraday's law is expressed in the form,

$\nabla \mathbf{XH} == \in_\mathbf{o} \frac{\partial \boldsymbol{E}}{\partial \boldsymbol{t}} = \frac{\partial D}{\partial t}$(xxi)

It can be interpreted that $\in_\mathbf{o} \frac{\partial \boldsymbol{E}}{\partial \boldsymbol{t}}$may not be viewed as the displacement current in Maxwell's theory as the fields ***E***and ***H*** described by (xiii) and (xx) are produced by magnetic current where no electric current is involved.

Finally, the set of equations used by Hertz in comparison to Maxwell’s field equations to develop his electromagnetic theory is represented as follows.

$\boldsymbol{B} = \mu_0 (\nabla \boldsymbol{X}\, \mathbf{I})$ $\Longrightarrow$ $\mathbf{H} = \nabla \boldsymbol{X}\, \mathbf{I}$ **(c)**

$\mathbf{E} = \nabla\, \mathbf{XI}_\mathbf{m}$**(d)**

$\mathbf{H} = \in_\mathbf{o} \frac{\partial \boldsymbol{Im}}{\partial \boldsymbol{t}}$ **(e)**

$\mathbf{E} = -\mu_\mathbf{o} \frac{\partial \boldsymbol{I}}{\partial \boldsymbol{t}}$**(f)**

These equations represent the behavior of electromagnetic field but without using the displacement current concept and hence no inconsistencies arise.

4. Experimental Studies & Displacement CurrentMeasurement

From above discussion it is clear that displacement current may be a way to converge regulations of Ampere’s and Faraday’s workinto a wave condition whose presence was confirmed by Hertz's disclosure. But despite the vital meaning of the displacement current in physical science, there is nosatisfactory explanation yet,of its quantitative measurementunder room conditions. Actually, a characteristic methodology must be developed to quantify the attractive field produced by the displacement current.

Accordingly, Bartlett and Corle [11]in theirexperimental attempt utilized a SQUID magnetometer as a probe andsafeguarded the capacitor by a superconductor. But the arrangement was not reasonable for the study hall because of its exploratory difficulties. Comparatively easier showing was introduced via Carver and Rajhel[12], though the proof wasn't altogether convincing.

To detect directly Maxwell's proposed displacement current in magneticfield, a trial has been demonstrated [13] using a straightforward experimental setup. The crucial trick is the use of a ferrite rod, which permits the induction coil to be placed outside the plate capacitor, substantially minimizing shielding concerns. Furthermore, frequency modulation amplification of the electric potential is used to minimize noise and demonstrate precisely thegeometry of the developed magnetic field, as well as its time-dependentpotentiality. This experimental setup is found to be simple,thoughrobust. Its accessibility of all key componentsas well as the conceptual simplicity, makes it appropriate for lecture hall presentation trials.But here the electric field of the capacitor and its coupling with the capacitance of the acceptance loophas been identified only, which is not sufficient to confirm the existence of Displacement Current.

Actually, experimental confirmation for the discovery of the displacement current must require the measurement of correct value of the magnetic field, its symmetry and corresponding dynamics. More importantly, the arrangement has to be made reasonable andin the real world the fundamental parts ought to be noticeable.

5. Conclusion

Maxwell's fundamental equations, which form the foundation of electromagnetic theory have been briefly reviewed in the context of displacement current. The role played by Hertz in confirming Maxwell’s theory and hence theacceptance of Maxwell's hypothesis have beenshown.Finally, set of equations representing electric and magnetic field using the concept of Hertz was discussed, though raising some questions on displacement current status and its experimental detection which might inspire other workers to concentrate for more extensive studies on this important topic to tackle the incompleteness and ambiguities in the explanation of displacement current.

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