



# Bidirectional Position control of the Prismatic joint for Motorized single link Robotic arm using Adaptive Super-Twisting Sliding Mode Control

Then Mozhi G. Banu Sundareswari M. and Dhanalakshmi K., Sr.\*

*(Department of Instrumentation and Control Engineering  
National Institute of Technology, Tiruchirappalli,  
Tamil Nadu, 620015, India)*

{Corresponding author's email: dhanlak@nitt.edu}

**Abstract - Dual layer adaptive super-twisting sliding mode control strategy (ASTSMC) designed for the position control of motor coupled flexible robotic arm is presented in this article. The system is modeled using the constraints computed for the DC motor using the nonlinear least-square (NLS) parameter estimation method. The flexible manipulator is modeled using lumped parameter model approach and the fixed extremity of the flexure is positioned at the desired point by the DC motor and the lead-screw linear actuation system. The super twisting algorithm and adaptive law is employed to control the system and the results are compared with PID controller and classical SMC. The final position accuracy is ensured, and the positioning performance is enhanced by ASTSMC controller design, and the simulated and experimental results show that the design of the proposed integrated mechanism and control system based on ASTSMC is effective in achieving precision in the placement of the flexure.**

**Keywords - manipulator; motorized assembly; super-twisting SMC; adaptive SMC; dual layer SMC.**

## INTRODUCTION

Automation is the scope of industrialization to transform the nature and economics of factory floors. The significant advancements in robotics lead to a new era of automation that allows machines to compete with or outperform humans. Automated robots and information technology are used for handling various processes in industries to replace humans. Most industrial applications like spray painting, loading-unloading, material handling, spot welding, etc. use robotic manipulators to follow a path to perform a task. In addition to DC motor, servo motor and stepper motor, some researchers have developed smart material based driving mechanisms for the

control of manipulators. Shape memory alloy based actuation and control of manipulators have been the recent area of research for building compact actuation mechanisms [1,2].

The construction employed for the sliding motion control of a single link robotic arm is a motor coupled system in which an electric motor drives a threaded screw to move the follower along the screw in the direction determined by the rotational direction of the motor. The angular rotation of the shaft of the motor drives the threads of the screw in the direction indicated by the controller. The DC motor incorporated in the application needs to be modeled for its parameters for accurate control. The mathematical modeling can be improved by computing the parameters involved in the electrical and mechanical dynamics of the motor from the measured behavior of the system. Several utilities used for estimating the values are frequency response method, algebraic estimation method as reported in [3], ordinary least squares method etc. The nonlinear least square (NLS) methodology is adopted as the optimization approach to minimize the error by estimating the parameters step by step in this work and the optimal electrical and mechanical features of the motor are computed.

The practical systems are nonlinear, but a set of linear dynamic equations can be formulated to describe the system. It is appropriate to use nonlinear analysis tools to control these systems efficiently. So nonlinear control scheme, SMC is developed and applied to the defined nonlinear system and implemented for tracking control. This scheme aims to maintain the control trajectory in the sliding surface. The problem with traditional position controllers for such systems is that their performance declines with uncertainties and when quicker trajectories are required. The application of a



variable structure (VS) controller tackles this challenge and the sliding mode control (SMC) and adaptive super-twisting sliding mode control (ASTSMC) are designed for the modeled system.

The flexible manipulator is viewed as an Euler-Bernoulli beam and the flexible robotic arm can be considered as a spring-mass-damper system with force acting axially along the beam. As represented in [4], the motion of the cantilever beam is described by the transfer function relating the beam displacement to the force acting on the beam. The integrated system must be able to follow the reference trajectories accurately by the designed closed-loop controller. The contribution is the use of a dual layer adaptive super twisting higher order SMC to generate the control input, which differs from other controller topologies. Unlike conventional SMC, this adaptive STSMC controller, includes several tuning parameters, for the generation of the continuous control signal that leads to the superior performance of the controller as presented by [5]. The single-link robotic arm is realized using the cantilever beam and the DC motor with an external linear actuation mechanism is employed for the position control of the beam. Simulated and experimental results demonstrate the efficacy of the proposed control strategy and the structural configuration.

*A. System Description*

The mechanical arrangement of the robotic arm comprises a single joint, single link manipulator and DC motor attached with rotary to a linear actuator to drive the position of the arm. A prismatic joint that enables single-axis sliding motion is preferred for positioning the arm and the physical embodiment of the prismatic joint on the rectangular cross-sectional beam is formed by rolling the bearings of the screw. A gear motor, a combination of a motor and gearbox is used as a rotary actuator. The gear head slows down the rotational speed while boosting the torque output. Lead screw mechanism provides rotary to linear actuation for the sliding movement for positioning the manipulator. Bidirectional adjustment is enabled by the lead screw that is built with fine positioners. The screws with fine threads enable precise positioning and the nut attached to the arm and the lead screw allow the travel of the manipulator. The displacement of the beam is detected by the laser displacement sensor (Keyence IL 100)

and is transmitted to the computer through DAQ card for the estimation of the position of the hinged point of the beam. The estimated position is compared with the desired position and error signal is generated. The designed control law provides appropriate control signal that is given to the motor driving unit that drives the motor for the proper positioning of the flexible manipulator.

The motorized actuation assembly generates the necessary movement for the bidirectional axial drive of the beam. The transfer function of the system motor must be identified for the simulation of the motorized system. The electrical and mechanical characteristics of the motor are determined by applying Kirchhoff's Voltage Law and Energy balance of the system as in (1) and (2).

$$V = R_a i + L_a \dot{i} + e_b \tag{1}$$

where,  $V$  is the input voltage (V),  $R_a$  is the armature resistance ( $\Omega$ ),  $L_a$  is the armature inductance (H),  $e_b = k_b \omega$  is the back EMF voltage (V),  $k_b$  is the back EMF constant and  $\omega$  is the angular speed of the motor (rad/s).

$$T = J \dot{\omega} + B \omega + \tau_l \tag{2}$$

where,  $T$  is the Motor Torque (N.m),  $J$  is the moment of inertia ( $\text{Kg.m}^2$ ),  $B$  is the damping coefficient ( $\text{Nm}/(\text{rad/s})$ ),

$\tau_l > k_t i$  is the load torque (N.m),  $k_t$  is the motor torque constant (N.m/A). The transfer function of the motor is derived from (1) and (2) and is given in (3).

$$\frac{\omega(s)}{V(s)} = \frac{k_t}{(L_a s + R_a)(J s + B) + k_t k_b} \tag{3}$$



The speed of the DC motor is measured by assuming the values of  $K_m, K_b, R_a, L_a, J, B$  with the initial set of values 0.1, 0.1, 0.5, 0.0001, 0.001, 0.01 respectively and simulated using MATLAB-Simulink. These simulated values are compared with the measured output of the DC motor, and the parameters are estimated using the nonlinear least square (NLS) method as depicted in Fig. 1. The motor is modeled by discarding higher-order terms to eliminate the effect of nonlinearities and substituting the identified parameters mentioned in Table I in (3).

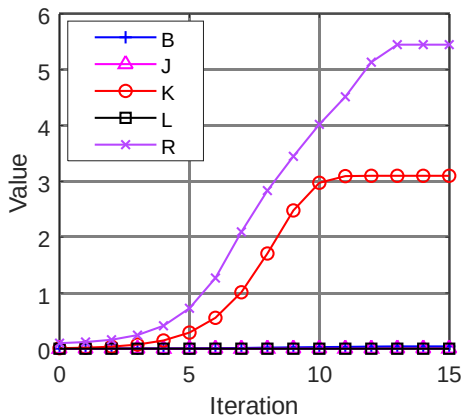


Fig. 1 Optimized parameters of DC motor using NLS method

TABLE I (DIMENSIONS AND PARAMETERS OF THE MOTORIZED ASSEMBLY)

Cantilever beam (Manipulator)		DC motor (Actuator)	
Material - Aluminium		Type - Gear	
Length, m	0.25	Maximum speed (rpm)	30
Width, m	0.02	Maximum input voltage (V)	12
Thickness, m	0.001	Armature resistance (Ω)	5.45
Density, kg/m <sup>3</sup>	2710	Armature inductance (H)	0.0002
Young's modulus, GPa	69	Moment of inertia (kg.m <sup>2</sup> )	0.0041
Mass, g	15	Damping coefficient (Nm/(rad/s))	0.0421
Stiffness, N/m	100	Back EMF constant (V/rad/s)	3.1
Damping coefficient, kg/s	2.45	Motor torque constant (N.m/A)	3.1

The manipulator resembles flexible beam and is considered as a homogeneous Euler-Bernoulli beam as studied in [6]. The single DOF cantilever system is governed by

Newton's second law of motion and the dynamics of the arm is given by (4),

$$M\ddot{\theta} + D\dot{\theta} + K\theta = F \tag{4}$$

where,  $M, D, K$  are the mass, damping and stiffness coefficients of the beam respectively, and  $F$  is the force applied to bring axial displacement  $X$  on the beam. These parameters are determined from the geometrical dimensions of the beam, density and Young's modulus.

The conversion cum transmission mechanism offers a mechanical advantage to the input of the robotic arm. The lead screw that converts the rotary motion of the DC motor to translational motion exhibits a mechanical advantage determined by the law of conservation of energy. i.e., the work done by the screw by the input force equals the work done on load and is given by (5).

$$F_i r = F_o l \tag{5}$$

where,  $r$  is the radius of the screw (0.25cm) and  $l$  is the lead or pitch of the screw (3 mm). The mechanical advantage of the lead screw is as in (6).

$$\frac{F_o}{F_i} = \frac{l}{r} \tag{6}$$

The mechanical advantage is determined as 5.23 and  $P_o > P_i > 6/34 P_i$ .

Considering the input force to the arm as  $F$ , and from the specifications of the beam presented in Table I, the transfer function of the arm is computed as in (7).

$$\frac{X(s)}{F(s)} = \frac{2}{s^2 + 2s + 1} \tag{7}$$

### B. Control Strategy

The control strategies like PID, SMC, and dual layer SMC are implemented for the control of the position of the robotic manipulator for the comparative analysis of the performance of the controllers. The control signal of the dominant, well known PID controller is given by (8).

$$\ddot{e}(t) + K_d \dot{e}(t) + K_p e(t) = \ddot{y}_d \tag{8}$$

$K_p, K_d, K_i$  are proportional, derivative, and integral gains and  $e(t)$  being the error signal.

Controller gains,  $K_p, K_i$  and  $K_d$  of PID controller is very sensitive to the load parameter but, the



control signal of variable structure controller easily tolerate continuous changes in load parameter and external disturbances.

The variable structure control, where a suitable output function of system states is assumed in which the control input appears in its first derivative, after which the control input is designed in discontinuous form such that the sliding manifold attracts the system states in finite time, and the system states slide on this manifold toward the origin. One of the basic nonlinear control approaches is Sliding mode control (SMC) that features noteworthy properties like easy implementation, precision, robustness, insensitive to external disturbances and uncertainties. This control drives the states of the system on to a specific surface called sliding surface. However, the limitation of this classical SMC is chattering and instability.

Sliding surface of classical SMC is given by,

$$s = \lambda e + \dot{e}, \quad \lambda > 0 \quad (9)$$

where,  $e(t)$  is the error signal,  $e(t) = y - y_{ref}$ .  $y$  is the observed output and  $y_{ref}$  is the reference signal.

The discontinuous control signal is added abruptly to reduce the error that causes chattering. The standard first order SMC uses  $sgn$  function that causes more chattering; so, this is replaced by  $sat$ ,  $\tan$  or sigmoid functions to overcome chattering effect. Another approach to tackle this issue is to use higher order SMCs. Super Twisting SMC is the unique SMC that converges the states of the system to the sliding surface and possess the ability of chattering reduction. The state trajectory of the STSMC twists and approaches the origin. Dual layer SMC that includes adaptive law and super twisting algorithm for disturbance and uncertainty rejection depicted in Fig. 2 is implemented in this work.

Global nonlinear sliding surface defined by [7] is as in (10).

$$s = \lambda e + \beta \int e dt \quad (10)$$

$$\dot{s} = \lambda \dot{e} + \beta e \quad (11)$$

$\lambda, \beta$  are positive constants. Continuous term of the control law that creates the sliding surface is derived from the above theory.

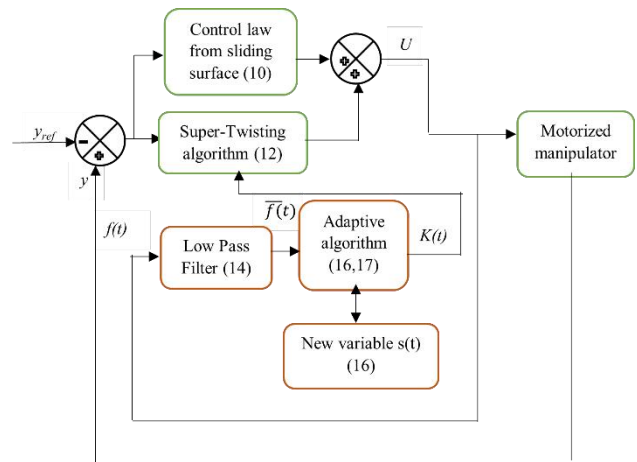


Fig. 2 Scheme of the ASTSMC control for the motorized Manipulator.

Super Twisting algorithm offers better control for systems with uncertainties with lesser control effort and forms the discontinuous term as presented in [8] and is represented in (12).

$$u = -\lambda_1 \dot{s} - \lambda_2 |s|^{2/3} \text{sgn}(s) - \lambda_3 \int s dt \quad (12)$$

$I_1(t) = \lambda_1 \int s dt$  and  $I_2(t) = K(t) \int s dt$ , where  $I_0$  is a positive constant and  $\lambda_1 > 3\lambda_2^{3/2}$ .  $\lambda_1$  is selected as 1.1 and  $\lambda_2$  equals 2.97. The control signal  $U$  is obtained by summing continuous and discontinuous control terms. The uncertainty of the system is as in (13)

$$\dot{s} = \lambda \dot{e} + \beta e \quad (13)$$

The uncertainty is estimated in real time by passing it to a low pass filter and the output of the filter is given by (14)

$$\dot{s} = \frac{2}{\tau} \int s dt \quad (14)$$

$RC = \tau$ , represents time constant of the filter.

$$\dot{s} = \frac{2}{\tau} \int s dt \quad (15)$$

The difference between the estimated value and the true value in (15) can be made small by choosing smaller  $\tau$ .

The adaptation law is incorporated in the design of the controller to represent safety margins. The new variable is chosen that includes adaptive control element  $K(t)$  as in (16).

$$\dot{K}(t) = \frac{2}{\tau} \int s dt \quad (16)$$

$$K(t) = k_0 + k(t), \quad k(t) = \frac{2}{\tau} \int s dt - r_1 + r_1 e^{-rt} \quad (17)$$



The adaptive gain  $K(t)$  can be determined as in (17),  $a$  and  $\varepsilon$  represent safety limits and  $0 < a < 1/b < 1$  and  $\forall k_0, r_0$  are positive scalars.

The gains  $l_1(t)$  and  $l_2(t)$  depend on  $K(t)$  and this parameter must be small as possible, but greater than the uncertainty  $f(t)$ . ASTSMC control uses several tuning parameters for eliminating uncertainties and disturbances for the precise control of the system.

TABLE II  
(TUNING PARAMETERS)

Controll er	$K_p$	$K_i$	$K_d$	$\lambda$	$\beta$			
PID	2.2	16.74	0.58	-	-			
SMC	-	-	-	0.34	5200			
ASTSMC	$\lambda_1$	$\beta$	$\tau$	$a$	$k_0$	$r_0$	$\gamma$	
		$l$		$b$				
	3.	2.	0.001	0.	0.	0.	0.01	4.7
	2	4		9	1	1		

C. Position Tracking by Controllers

The controllable robot manipulator link is connected by a prismatic joint permitting translational displacement and the measurement on the position of the arm is detected by the laser displacement sensor that is placed at the feedback loop of the control system as shown in Fig. 3. The beam length of 15 cm is chosen as position 0 (equilibrium point) and to achieve linear position towards the right (position > 0) the control signal operates the DC motor in a clockwise direction whereas to attain position towards left (position < 0), the motor is activated in the counterclockwise direction.

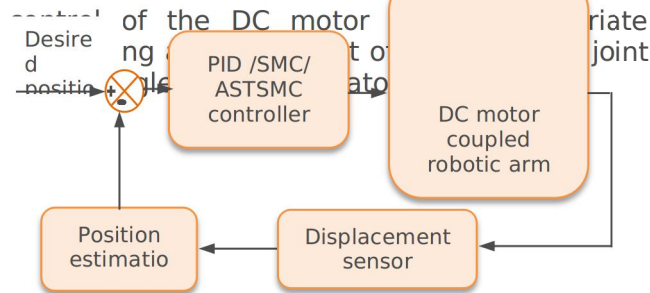


Fig. 3 Control system for the motorized robotic arm.

The control schemes are implemented through MATLAB - Simulink and demonstrated on the

flexural beam that represents the industrial robotic arm. ASTSMC controller is realized using MATLAB/Simulink environment. The desired position of the beam is set initially at 2 cm and the controllers are tuned using trial and error method of tuning to achieve the target position; the tuning parameters are as in Table II.

The response of the system for the fixed step is shown in Fig. 4 (a) and the performance of the system is depicted in Table III. The system for varying reference input is checked for its performance and is as in Fig. 4 (b).

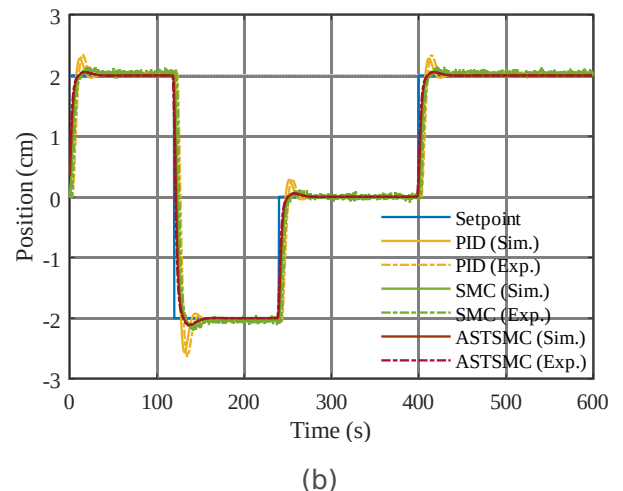
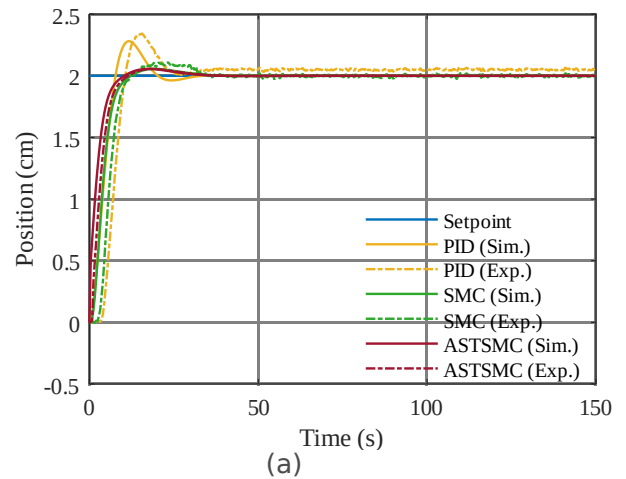


Fig. 4 Response of the system: (a) for fixed-step input (b) for varying step input

TABLE III  
(PERFORMANCE PARAMETERS)

Performance	PID	SMC	ASTSMC
Rise time (s)	10.7	7.8	7.4
Settling time	40.16	38.7	37.8



(s)				
Peakovershoot	25.4	6.2	4.4	
(%)				
Steady state error	0.054	0.006	0.0042	

*Transactions of the Institute of Measurement and Control*, vol. 38, pp.602-617,2017.

[7] Mobayen, Saleh Tchier, Fairouz Ragoub and Lakhdar, " Design of an adaptive tracker for n-link rigid robotic manipulators based on super-twisting global nonlinear sliding mode control," *International Journal of Systems Science*.vol. 48, pp.1-44,2017.

[8] Edwards, C. Shtessel and Yuri, "Adaptive Dual Layer Super-Twisting Control and Observation," *International Journal of Control*.vol. 89,pp.1759-1766,2016.

Simulated and experimental results show the legitimacy of this control strategy. The validation of the abilities of the ASTSMC control is assured by simulated and experimental results. The ASTSMC control shows remarkable performance when compared to other controllers in terms of rise time, settling time, peak overshoot, and steady state error.

#### D. Conclusion

The DC motor and the flexible manipulator as a lumped parameter model has been derived and analyzed. The performances of PID, classical SMC and ASTSMC for the position control of a single link industrial manipulator have been demonstrated. The ASTSMC control strategy that involve adaptive gain, outperforms the traditional PID controller and classical SMC in terms of tracking performance. The rotary actuator modeled using the nonlinear least-square parameter estimation method and the modeled flexible arm is tested experimentally to validate its performance and the ASTSMC controller guarantees the improved performance of the system. The response can further be improved by tuning the parameters of the controller using appropriate optimization techniques.

#### REFERENCES

[1] Banu Sundareswari M, Then Mozhi G and Dhanalakshmi K."Angular control of differential shape-memory alloy spring actuator for underactuated dynamic system," *Journal of Vibration and Control*. September 2021.

[2] Josephine Selvarani Ruth D., Sunjai Nakshatharan, and Dhanalakshmi Kaliaperumal, "Differential resistance feedback control of a self-sensing shape memory alloy actuated system," *ISA transactions*, vol. 53, pp.289-297,2013.

[3] J. Beccedas, G. Mamani, and V. Feliu. "Algebraic parameters identification of DC motors: methodology and analysis," *International Journal of Systems Science*, vol.41,pp. 1241-1255,2010.

[4] C. Sun, W. He and J. Hong, "Neural Network Control of a Flexible Robotic Manipulator Using the Lumped Spring-Mass Model," *IEEE Transactions on Systems, Man, and Cybernetics*,vol.47, pp.1863-1874,2017.

[5] Biricik H., Komurcugil H. Ahmed and E. Babaei. "Super Twisting Sliding-Mode Control of DVR With Frequency-Adaptive Brockett Oscillator," *IEEE Transactions on Industrial Electronics*,vol.68, pp.10730-10739,2021.

[6] Castillo-Berrio CF, Engin SN and Feliu-Batlle V. "A study on the tip tracking control of a single flexible beam,"