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OPTIMUM SIZE OF RADIUS of CURVATURE FOR A TRAPEZOIDAL CHANNEL TO MINIMISE HYDRAULIC LOSS

Abstract

Optimum value of the radius of curvature to be provided to join the bed of a trapezoidal channel with side slopes has been worked out, to minimize the head loss in the channel. It has been found to be equal to 0.4 R, where R is the mean hydraulic radius of the channel, for all values of side slopes. Provision of curves with a radius equal to the depth of the channel has been found to increase the head loss instead of decreasing it, thus defeating the very purpose for which it is provided.

Introduction

In lined canals and hydel channels with trapezoidal cross-section, usually circular curves are provided to join the bed with the side slopes. The objective of these curves is to smooth the flow of water and hence reduce the hydraulic head loss in the channel. The radius of the curves is fixed on ad-hoc basis. In India, for more than a century, the practice is to provide a radius equal to the water depth in the channel. In this paper, the optimum value of the radius of curvature has been worked out, to minimize the hydraulic head loss in the channel. This optimization becomes of paramount importance in the case of hydel channels and tail race channels of hydro power plants, as each millimeter of head saved, can result in thousands of more units of power generation. It has been found that the century old practice of providing curves equal to the water depth of the channel, actually increases the head loss, instead of decreasing it, thus defeating the very purpose for

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which it is provided.

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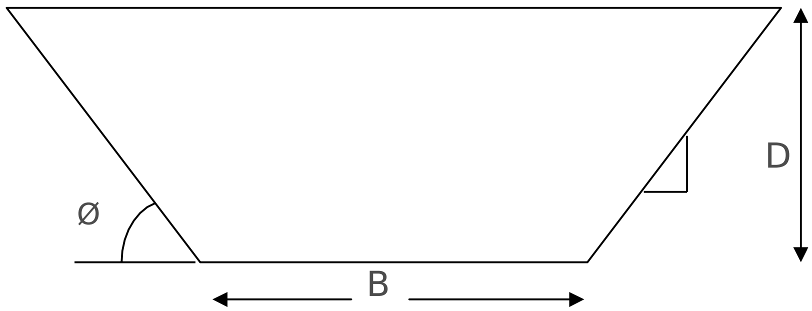
DERIVATION OF MATHEMATICAL RELATION:-

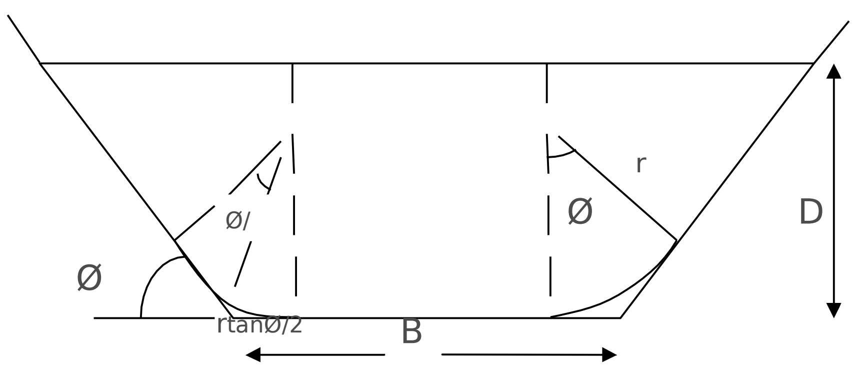
Consider a trapezoidal channel of width B and depth D with inclination angle of the sides to the base of the channel as Ø and having a curve of radius r at the bottom 2 corners.



Area (A) of trapezoidal channel $A = BD + D^2 \cot Ø$

Wetted perimeter(P) of trapezoidal channel $P = B + 2D\,(1+\cot^2 Ø)^{1/2}$

$= B + 2D \operatorname{cosec} Ø$



Area of trapezoidal channel with curve (A_1)

$$A_1 = BD + D^2 \cot Ø - 2x \left[\frac{2 \times r \times r}{2} \tan Ø/2 - \Pi r^2 \times Ø / 2\Pi \right]$$

$$= BD + D^2 \cot Ø - 2\, r^2 \tan Ø/2 + 2r^2 \times Ø / 2$$

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Wetted perimeter of trapezoidal channel with curve(P_1) P_1= B + 2D cosecØ - 2x [2 x rtanØ/2 - 2Πr xØ /2Π]

= B + 2D cosecØ - 4 x rtanØ/2 + 4r xØ /2 δA= A-A_1 = 2 r^2 tanØ/2 - 2r^2 xØ /2

= 2r^2[tanØ/2 - Ø /2]

δP= P-P_1 = 4 r tanØ/2 - 4r xØ /2

= 4 r [tanØ/2 -Ø /2]

Velocity (V) of fluid flowing through the trapezoidal channel

$V = \frac{Q}{A}$ (Q is discharge of fluid)

By Manning’s equation

$V = \frac{1}{n} R^{2/3} s^{1/2}$ {where n= rugosity coefficient

= k*ḃ· 5⁚4*b ⁚21⁚4

s_1 = ·ṙḃ· ⁚ᐊḃ· 5⁚4 / b ⁚ᐊb 21⁚4

·ḷ 2⁄3

> J⁚b / b ᵇ ⁚3⁄4

s =

J3⁚b 3 / b 3 ᵇ ⁚5⁄4

= ·ṙᵇ ⁚5⁚4 / b 3

(where k=n^2Q^2)

= ·ṙ / b 21/4 *ḃ· 5⁚4

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longitudinal slope}

R

=

H
y
d
r
a
u
l
i
c

R
a
d
i
u
s

=

A
/
P

s
=

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As δP and δA are small compared to P and A,

$$\therefore s_1 = \frac{\cdot ṙ\, ḃ\cdot\, 5∷4\ \frac{5}{4}ḃ\cdot\ 2∷4ⲴḃΓ}{ƅ\ 21/4\ \frac{21}{4}ƅ\ 8∷4Ⲵƅ}$$

$$= \cdot ṙ\, ḃ\cdot\ 5∷4\ \vdots \frac{5}{4} ḃ\cdot\ 2/4 Ⲵ ḃ\cdot\ ƅ\ \vdots 21/4,\ \frac{21}{4} ƅ\ \vdots 24/4 Ⲵ ƅ$$

$$= \cdot ṙ\,ḃ\cdot\ 5∷4ƅ\ \vdots 21/4\ 2\ \vdots \frac{5}{4}\frac{Ⲵḃ\cdot}{ḃ\cdot},\ \frac{21}{4}\frac{Ⲵƅ}{ƅ}$$

$$= \cdot \dot{l}\ 2\ \vdots \frac{5}{4}\frac{Ⲵḃ\cdot}{ḃ\cdot},\ \frac{21}{4}\frac{Ⲵƅ}{ƅ}$$

$$δs = s - s_1 = \cdot\dot{l}\ \frac{5}{4}\frac{Ⲵḃ\cdot}{ḃ\cdot}\ \vdots \frac{21}{4}\frac{Ⲵƅ}{ƅ}$$

$$\frac{δ\cdot\dot{l}}{\cdot\dot{l}} = \frac{5}{4}\frac{Ⲵḃ\cdot}{ḃ\cdot}\ \vdots \frac{21}{4}\frac{Ⲵƅ}{ƅ}$$

For $\frac{δ\cdot\dot{l}}{\cdot\dot{l}}$ to be maximum, $\frac{J̈\ r̊}{J̈\ l\cdot} > 1$ where $r̊ > \frac{Ⲵ\cdot\dot{l}}{\cdot\dot{l}}$

$$δA = 2 r^2 [\tan Ø/2 - Ø/2]$$

$$δP = 4 r [\tan Ø/2 - Ø/2]$$

$$\therefore \frac{Ⲵ\cdot\dot{l}}{\cdot\dot{l}} = y = \frac{5}{4}\frac{5l\cdot\ \backslash \tan Ø/2 - Ø/2\ ^\wedge}{ḃ\cdot}\ \vdots \frac{21}{4ƅ}\left(2 r^2 [\tan Ø/2 - Ø/2]\right)$$

$$\frac{J̈\ r̊}{J̈\ l\cdot} = \frac{27}{4ḃ\cdot}[\tan Ø/2 - Ø/2] - \frac{51l\cdot}{4ƅ}[\tan Ø/2 - Ø/2] = 0$$

$$\therefore \frac{27}{4ḃ\cdot} = \frac{51l\cdot}{4ƅ}$$

$$r = \frac{3ƅ}{6ḃ\cdot} = 0.4 * \frac{ƅ}{ḃ\cdot}$$

$$= 0.4 * R$$

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Illustration

Assume a trapezoidal channel with Discharge (Q) =400 cumecs, B=20m, Depth of channel= 6m, Side Slope = 1.5:1

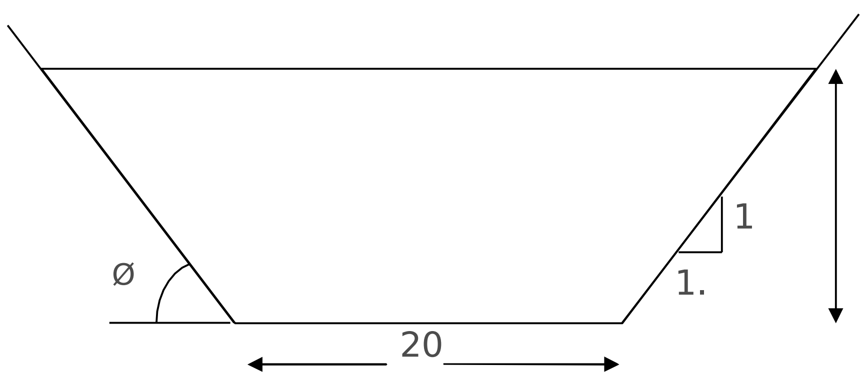
Using rugosity coefficient (n)

= 0.018 Bed Width of the

channel (B) = 20m Depth of

the channel (D) = 6m

Case I: Without Curve



Area (A) of trapezoidal channel

A $= \frac{1}{2}(20 + 38)6$ = 174

Wetted perimeter(P) of trapezoidal

channel P = B + 2D cosecØ

= 20+ 2x6x1.8028

P = 41.6336

R = / = 4.1793

V $= \frac{1}{n} R^{2/3} s^{1/2}$

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$$Q = A\frac{1}{n}R^{2/3}s^{1/2}$$

$$400 = 174 \times \frac{1}{0.018} 4.1793^{2/3}s^{1/2}$$

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$s = 2.5435 \times 10^{-4}$

Case II: With radius of Curve, r=D

i.e. r=6m

cotØ = 1.5 ; Ø=33.6901º

$A = BD + D^2 \cot Ø - 2r^2 \tan Ø/2 + 2r^2 \times Ø/2$

$A = 20\times6 + 6^2 \times 1.5 – 2\times 6^2 \times \tan(33.6901/2) +$

$2 \times 6^2 \times 0.588/2$ A = 173.3664

P=B + 2D cosecØ - 4 x rtanØ/2 + 4r xØ /2

P = 20 + 2x6x1.8028 – 4x6x0.3028 +

4x6x0.588/2 P = 41.4224

R = / = 4.1853

$Q = A \frac{1}{n} R^{2/3} s^{1/2}$

$400 = 173.3664 \times \frac{1}{0.018} 4.1853^{2/3} s^{1/2}$

$s = 2.5572 \times 10^{-4}$

Case III: With radius of Curve, r=0.4R where R = Hydraulic Mean Radius cotØ = 1.5 ; Ø=33.6901º

$A = BD + D^2 \cot Ø - 2r^2 \tan Ø/2 + 2r^2 \times Ø/2$

$A = 20\times6 + 6^2 \times 1.5 – 2\times r^2 \times \tan(33.6901/2) + 2 \times r^2 \times 0.588/2$ $A = 174 – 2r^2(0.3028 - 0.294)$

$A = 174 – 2(0.4R)^2(8.8 \times 10^{-3}) = 174 – 2.816 \times 10^{-3} R^2$

P=B + 2D cosecØ - 4 x rtanØ/2 + 4r x Ø /2

P = 20 + 2x6x1.8028 – 4 x r [0.3028 - 0.294]

$P = 41.6336 – 4(0.4R)(8.8 \times 10^{-3})$

$P = 41.6336 – 14.08 \times 10^{-3} R$ R = A/P

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$R = 174 - 2.816 \times 10^{-3} R^2 / 41.6336 - 14.08 \times 10^{-3} R$

$R = 4.18405$

$A = 174 - 2.816 \times 10^{-3} R^2 = 173.9507$

$$Q = A \frac{1}{n} R^{2/3} s^{1/2}$$

$$400 = 173.9507 \times \frac{1}{0.018} 4.18405^{2/3} s^{1/2}$$

$s = 2.5411 \times 10^{-4}$

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	Longitudinal Slope (S)
Without Curve	2.5435×10^{-4}
Radius of Curve, r =D	2.5572×10^{-4}
Radius of Curve, r =0.4R	2.5411×10^{-4}

Assuming length of hydel

channel=10kms For Radius of

Curve, r =D and r=0.4R

Head saved = $10000 \times (2.5572 \times 10^{-4} - 2.5411 \times 10^{-4}) = 0.0161$ = 16.1mm

Extra power generated = $400 \times 9.8 \times \frac{16.1}{1000} \times 1000$

= 63.112 kW

Considering load factor = 60%

Extra units of power generated per annum = 63.112 x 0.6 x 24 x 365 = 331716.67 kWh = 3,31,716 units, say 3.3 Lakh units

Therefore, provision of Radius of Curve, r =0.4R in place of r=D saved 3.3 Lakh units of power per annum.

Conclusion.

It has been found that the optimum radius of curvature to minimize the hydraulic head loss, is equal to 0.4 R where R is the hydraulic mean radius of the channel, for all values of side slopes. It has also been found that provision of a radius equal to the water depth in the channel, actually increases the head loss, instead of decreasing it, and thus defeats the very purpose for which it is provided. The practice of providing a radius equal to the depth of the channel must be discontinued forthwith, and instead, a radius equal to 0.4 R should be provided. Provision of a curve with radius equal to 0.4R becomes of paramount importance in the case of Hydel channels and tail race channel of power plants, as each millimeter of head saved can result in

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thousands of more units of power production.