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Optimization of straight fins with different geometries using PYTHON: A comparative study

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Abstract:

Increasing the available surface area for heat transfer is the common practice to enhance the heat transfer rate, where coefficient of heat transfer is low. Fins are commonly used to achieve increased heat transfer area. One-dimensional analysis of straight fins with different geometries such as rectangular, triangular and Parabolic is carried out using various gradient free optimization search methods. Optimum fin thickness for different geometries was estimated by keeping the space available for fin as a constraint. Codes are generated for various gradient free numerical optimization search methods in Python. The optimum fin thickness for each of the fin geometries are computed with different gradient free search methods.

Keywords: fins; optimization; Python; heat transfer analysis; gradient free search.

Introduction:

The convective rate of dissipation is governed by Newton's law of cooling, Q = hAs(Ts-Ta). From this equation, it is observed that whenever heat transfer coefficient is low, the practical solution to enhance the heat transfer rate is by increasing area of heat transfer. Fins[1,2] are the normally used extended surfaces to increase heat transfer from various thermal sources. Fins are used as heat sink for various electronic packages, refrigeration, vehicle radiators and engines and various heat exchangers. Since massive amount of heat generated by these devices can harshly affect their performances and can cause harm to their parts, the necessity for efficient fins with minimum possible size and weight has been increased. The heat transfer rate can be improved by increasing the fin dimensions, incorporating complex shapes, use of high thermal conductivity materials, increasing heat transfer coefficient and decreasing temperature of ambient fluid. Among these, use of materials with high thermal conductivity may not be cost effective. Also, increasing heat transfer coefficient and decreasing ambient temperature is impractical.

Florent Bunjaku et al. have carried out thermal optimization of geometric parameters of rectangular and triangular fins and numerical simulation of fin models was carried out using ANSYS/Fluent software[3]. Akshendra Soni studied steady-state thermal performances of plate-fin, pin-fin and elliptical heat sinks with vertical base plate natural convection and the 3D analysis was carried out using ANSYS software[4]. Hyung Suk Kang optimized concave parabolic fin for fixed fin volume using a two-dimensional analytical method[5]. Hyung Suk Kang et al. used a new approach for optimization for the design of an annular trapezoidal fin[6]. They optimized heat loss, fin tip radius, and fin base height for fixed fin volume as a function of the ratio of convection characteristic numbers, fin shape factor, dimensionless fin volume, dimensionless fin base radius, and convection characteristic number. Antonio Campo et al. carried out heat transfer analysis analytically by solving approximately the quasi-1D heat conduction equation which is in the form of generalized Airy equation governing the annular fin of hyperbolic profile[7]. Kabir Bashir Shariff et al. carried out numerical study on the effects of fins in the performance of car radiator under the atmospheric temperature[8]. The car radiator model was modelled in SolidWorks and the model was analysed in ANSYS with water as the cooling fluid. Aissa Yousfi et al. carried out forced convection heat transfer and fluid flow characteristics in a pin fin heat sink[9]. They proposed a new pins design and also validated the design with three configurations such as cylindrical, rectangular and square. Ugur Akyol et al. conducted an experimental study to investigate the heat transfer and friction loss characteristics in a horizontal rectangular channel having attachments of hollow rectangular profile fins over one of its heated surfaces[10].

Selection of geometry and optimization methods:

Selection of fin material, fin geometry and optimization of fin dimensions are the factors to be considered in the design of fins. The fin material selected for the present study was Aluminium. Fins are available in various geometries. Geometries such as rectangular, triangular convex and concave parabolic straight fin are considered for analysis. The same temperature at the fin base were taken for all the fin geometries. It is obvious that as the fin dimensions increases, convective heat transfer rate also increases. But it is impractical to increase the dimensions beyond a limit as the space available to accommodate fins is a constraint. In this case study, thermal analysis and optimization were carried out with fixed fin width. Considering given fin volume, the variation of length goes through a maximum total heat dissipation from the fin surface. Code for various gradient-free search methods developed in Python was used for 1D optimization of the fin with different geometries. One dimensional analysis is widely used to understand the complex systems and interpret the system behaviour. The modelled equation is highly non-linear and so, analytical solution for optimum fin length is difficult. Thus, gradient free optimization search methods like Fibonacci, Golden section and Dichotomous search methods were used for finding the optimum fin length by keeping the space available for fin as a constraint.

Selection of platform for optimization coding:

Python being one of the most powerful and popular high-level programming languages, freely available and used in various engineering fields, the codes for present work are generated in Python. Anaconda Individual Edition, the most popular Python distribution was used for developing the code[11]. The optimization code was developed and run in Python's own development environment, IDLE (Integrated Development and Learning Environment).

Mathematical model:

1-Dimensional optimization study was carried out with following assumptions.

- Steady state heat flow in and out of the fin.
- Material is isotropic and temperature invariant thermal conductivity ‘k’.
- Temperature of ambient fluid is constant.
- Heat transfer coefficient is uniform over entire surface.
- The fin width and thickness are too long compared to its length so that 1D analysis is valid.
- Only mode of heat transfer between fin and surrounding is convection.
- Convection at the tip of the fin is also considered.
- No heat generation within the fin

Mathematical model of the system is described by

$$\frac{d}{dx}\left[A_c\frac{dT}{dx}\right]-\frac{hP}{kA_c}(T-T_a)=0 \qquad (1)$$

The equation is linear homogenous 2nd order differential equation with constant coefficients.

Boundary conditions:

Constant temperature was taken at the fin base. Convective boundary condition was taken at the entire fin surfaces and also at exposed surface of the fin base. No heat flux is taken at other two surfaces of fin base.

$T=T_w$ at $x = 0$

$$-kA_c\frac{dT}{dx}=hA_c(T-T_a)\ \ at\ x=L \qquad (2)$$

For rectangular fin the above equation becomes

$$\frac{d^2T}{dx^2}-m^2(T-T_a)=0 \qquad (3)$$

For rectangular fin, P=2(w+t) and Ac= wt.

On solving the modelled equation with the above boundary conditions, the total heat transferred is obtained as,

$$q_{tot}=\sqrt{(hPkA_c)}(T_w-T_a)\frac{\tan h(mL)+\frac{h}{mk}}{1+\frac{h}{mk}\tan h(mL)} \qquad \text{where}, m=\sqrt{\frac{hP}{kA_c}}$$

--------(4)

The equation involves hyperbolic trigonometric function.

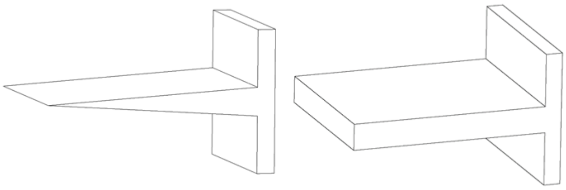

Fig. 1. (a) Triangular fin; (b) Rectangular fin

Triangular fin.

For triangular fin the modelled equation becomes

$$x\frac{d^2(T-T_a)}{dx^2}+\frac{dT}{dx}-m^2(T-T_a)=0 \qquad (5)$$

On solving the modelled equation with the boundary conditions, the total heat transferred is obtained as,

$$q_{tot}=\frac{2hw}{m}(T-T_a)\frac{I_1(2mL)}{I_0(2mL)} \quad \text{where}, m=\sqrt{\frac{2h}{tk}}$$

(6)

The equation is in terms modified Bessel functions of first kind.

1.1. Concave parabolic fin

For concave parabolic fin, the expression for total heat transferred is obtained as,

$$q_{tot}=\frac{ktw}{2L}(T-T_a)\left[-1+\sqrt{1+(2mL)^2}\right] \qquad (7)$$

$$\text{where}, m=\sqrt{\frac{2h}{tk}}$$

Convex parabolic fin

For convex parabolic fin, the expression for total heat transferred is obtained as,

$$q_{tot}=ktwm(T-T_a)\frac{I_{2/3}\left(\frac{4}{3}mL\right)}{I_{-1/3}\left(\frac{4}{3}mL\right)} \quad \text{where}, m=\sqrt{\frac{2h}{tk}}$$

(8)

The equation is in terms modified Bessel functions of first kind.

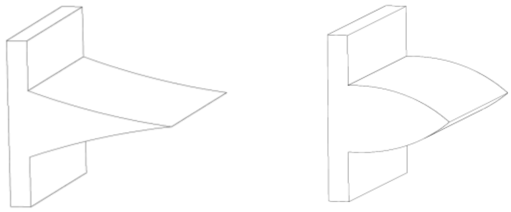

Fig. 2. (a) Concave fin; (b) Convex fin

Solution methodology:

Since all the equations for q_{tot} are highly non-linear and involves special mathematical functions, analytical optimization methods can't be applied. So, any of the numerical method has to be used. In this paper, gradient-free search methods such as Fibonacci, Golden section and Dichotomous search methods are used to find the optimum fin length at which the q_{tot} is maximum. All the three methods are sequential search methods in which the result of any iteration influences the location of the subsequent iteration. In the dichotomous search, two search points are located as close as possible at the center of the interval of uncertainty. Based on the relative values of the objective function at the two points, almost half of the interval of uncertainty is eliminated. The Fibonacci method makes use of the sequence of Fibonacci numbers. The optimum point is located within the final interval of uncertainty. In the golden section method, a fixed ratio, i.e., golden ratio (3 - √5/2) is used to calculate search points. The effectiveness of an elimination method can be measured in terms of the ratio of the final and the initial intervals of uncertainty.

Since, Python is free and open-source programming language and has a simple and flexible code structure, the codes of three search methods are developed in Python. For the rectangular fin, hyperbolic tangent function was accessed through Python's math module [12,13]. For the triangular and convex parabolic fins, Bessel functions are accessed through Python's Scipy library.

The results from all the three methods are compared based on final to initial interval reduction ratio within 2%. In this case study, the following values are taken for geometrical parameters. Fin width, W = 1 m, thickness, t = 0.01 m, volume, V = 5×10^{-4} m^3. The properties and boundary conditions are taken as average thermal conductivity of fin material, k = 160 w/m k, average value of heat transfer coefficient, h = 120 W/m^2K, Tw = 150^0C an Ta = 25^0C. Optimization is carried out based on 2% interval reduction.

Results and Discussions:

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The total heat transferred through the fins were calculated for all the geometries using above equations. Fig (1) to (4) shows the total heat transferred by varying the fin length from 0.025 m to 0.2 m by keeping the volume of fin as constant. From Fig (1) to (4), it can be seen that concave parabolic fin gives the maximum heat loss with available fin volume. Form the figures it can be depicted that each curve passes through a maximum heat loss value for a particular fin length. These optimum values were calculated using codes developed in Python 3.7 based on Fibonacci, golden section and Dichotomous algorithms. The developed codes are validated with some benchmark functions and the results are shown in table-1. The optimum of these bench mark functions was calculated analytically. The optimum values were also calculated numerically using codes developed in Python. From table-1, it can be seen that the values obtained from Python codes are well matched with analytical solution [14] and thus validated the code.

For the optimization of fin length, the objective functions consist of complex mathematical functions such as hyperbolic and Bessel functions. Since analytical solutions are complex, numerical methods are chosen and used the codes developed in Python based on Fibonacci, Golden section and Dichotomous algorithms[15]. The results obtained are listed in Table 2. Optimization is carried out based on 2% interval reduction. For all the four geometries the optimum length and corresponding heat transfer values obtained by different numerical methods are almost equal. Thus, all the three numerical methods are well suited for finding optimum fin dimensions. From the table it can be depicted that by keeping fin volume constant, the rate of heat loss is maximum for concave parabolic profile. Also, for constant fin volume, minimum fin length corresponds to rectangular profile.

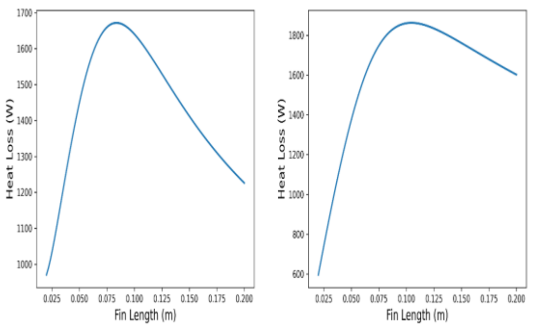

Fig. 3. (a) Heat loss vs fin length in rectangular fin; (b) Heat loss vs fin length in Triangular fin

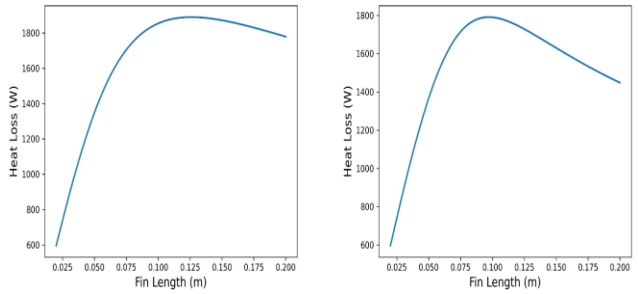

Fig. 4. (a) Heat loss vs fin length in Concave fin; (b) Heat loss vs fin length in Convex fin.

Table-2. Comparison of optimum fin length

	Fibonacci	Golden section	Dichotomous
Rectangular	L_{min} = 0.0853 m Q_{max} = 1671.34 w N_{itr} = 9	L_{min} = 0.0835 m Q_{max} = 1671.72 w N_{itr} = 9	L_{min} = 0.0833 m Q_{max} = 1671.71 w N_{itr} = 8
Triangular	L_{min} = 0.1062 m Q_{max} = 1861.86 w N_{itr} = 9	L_{min} = 0.1050 m Q_{max} = 1862.08 w N_{itr} = 9	L_{min} = 0.1041 m Q_{max} = 1862.08 w N_{itr} = 8
Concave parabolic	L_{min} = 0.1278 m Q_{max} = 1889.75 w N_{itr} = 9	L_{min} = 0.1260 m Q_{max} = 1889.88 w N_{itr} = 9	L_{min} = 0.1270 m Q_{max} = 1889.84 w N_{itr} = 8
Convex parabolic	L_{min} = 0.0953 m Q_{max} = 1790.87 w N_{itr} = 9	L_{min} = 0.0973 m Q_{max} = 1791.30 w N_{itr} = 9	L_{min}= 0.0978 m Q_{max} = 1791.25 w N_{itr} = 8

Conclusions:

In this study, one-dimensional numerical optimization of straight fins with different geometries such as rectangular, triangular, concave and convex parabolic profile is carried out. Various gradient free optimization search methods like Fibonacci, Golden section and Dichotomous search methods were used for finding the optimum fin length by keeping the volume available for fin as a constraint. The codes were developed in Python and validated with benchmark functions. Finally, the codes were used to optimize the fin length of all the four geometries based on specific final to initial interval ratio. From the obtained optimization result, it can be concluded that all the three numerical methods are well suited for finding optimum fin dimensions. The study revealed that Python can be effectively used to optimize engineering systems that models to complex mathematical equations.

Abbreviations:

Nomenclature

Q Heat transfer rate, [W]

H Heat transfer coefficient, [W/(m^{2}K)]

As Surface area, [m^2]

Ts Surface temperature, [K]

Ta Ambient Temperature, [K]

Ac Cross sectional area, [m2]

P Perimeter, [m]

K Thermal conductivity, [W/(mK)]

W Fin width, [m]

T Fin thickness, [m]

L Fin length, [m]

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